Cube Root of x

Show that for any non-zero starting point x_0, Newton's method will never find the exact value x for which $x^3 = 0$.

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$x^3 = 0$, $x_0 \neq 0$

$f(x) = x^3 = 0$ $\qquad$ $f'(x) = 3x^2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0^3}{3x_0^2}$$

$$= x_0 - \frac{x_0}{3}$$

$$= \frac{3x_0 - x_0}{3}$$

$$= \frac{2}{3}x_0$$

$$\Rightarrow x_{k+1} = \frac{2}{3}x_k$$

$$= \frac{2}{3}\left(\frac{2}{3}x_{k-1}\right)$$

$$= \frac{2}{3}\left(\frac{2}{3}\right)^{k-1}x_0$$

$$= \left(\frac{2}{3}\right)^k x_0$$

$x = 0$, $x_{k+1} = \left(\frac{2}{3}\right)^k x_0$.

$\Rightarrow$ For $x_{k+1} = x = 0$,

then $\left(\frac{2}{3}\right)^k x_0 = 0$.

$\Rightarrow \left(\frac{2}{3}\right)^k = 0$ or $x_0 = 0$

But $\left(\frac{2}{3}\right)^k \neq 0$, if $x_0 \neq 0$,

$\therefore$ $x_{k+1} \neq 0$ for any k. $\square$